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ATOMS, MOLECULES, OPTICS

Inelastic Processes in the Interaction of an Atom with an Ultrashort Electromagnetic Pulse

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Abstract—Electron transitions occurring during the interaction of a heavy relativistic atom with a spatially inhomogeneous ultrashort electromagnetic pulse are considered by solving the Dirac equation. The corresponding transition probabilities are expressed in terms of known inelastic atomic form factors, which are widely used in the theory of relativistic collisions between charged particles and atoms. By way of example, the inelastic processes accompanying the interaction of ultrashort pulses with hydrogen-like atoms are considered. The probabilities of ionization and production of a bound–free electron–positron pair on a bare nucleus, which are accompanied by the formation of a hydrogen-like atom in the final state and a positron in the continuum, are calculated. The developed technique makes it possible to take into account exactly not only the spatial inhomogeneity of an ultrashort electromagnetic pulse, but also the magnetic interaction. *© 2005 Pleiades Publishing, Inc.*

1. INTRODUCTION

The interaction of atoms with ultrashort electromagnetic pulses with a duration shorter than the characteristic atomic periods of time has become an object of investigation only recently. A new trend, viz., the physics of attosecond pulses (1 attos = 10^{-18} s), has been developed. The possibility of detection, generation, and application of attosecond pulses was discussed by many authors engaged in experimental and theoretical studies. The state of the art by the beginning of 2004 and the corresponding references are given in reviews [1–3] (see also several later publications [4–20]). The increased interest in the physics of ultrashort pulses is associated not only with modern tendencies in designing more powerful lasers and generation of ultrashort pulses [21], but also with the advances made in heavy-ion accelerator technique since the fields produced by relativistic and ultrarelativistic charged particles are close in properties to the field of a light wave. For example, in experiments [22] (see also [23–26]), double and single ionization of a helium atom by an impact of a uranium U^{92+} ion with an energy of 1 GeV/nucleon was studied and a ultrastrong pulse ($I > 10^{19}$ W/cm^2) with a duration on the order of 10^{-18} s was simulated. It is extremely difficult to obtain such parameters of an electromagnetic pulse by other available methods. For example, the observation of pulses with a duration of a few femtosecond was reported in [21], representing almost thirty years (up to 2000) of evolution in the physics of ultrashort laser pulses and technological achievements in the field of generating such pulses. Thus, collision experiments in fact offer the only possibility of simulating ultrashort pulses with a duration comparable to or smaller than the characteristic atomic time $\tau_a \sim 10^{-17}$ s. Collision experiments also provide the subsequent opportunity for direct observation of the interaction between atoms and an ultrashort electromagnetic pulse. In a comparatively recent experiment [27], multiphoton production of pairs by an ultrarelativistic electron moving with a relativistic factor of $\gamma \sim 10^5$ through an ultrastrong laser field was observed; in this case, in the rest system of the electron, the laser field frequency and strength increased approximately by a factor of γ. In recent theoretical works [28, 29], the processes of multiphoton pair production during collisions of bare ultrarelativistic nuclei with high-intensity laser radiation were considered and the possibility of conducting the corresponding experiments on modern accelerators was also noted. Thus, during the collision (interaction) of a target atom moving with a relativistic energy (or a partly stripped atom, viz., a structural ion with a certain number of electrons in its shells) with an ultrashort electromagnetic pulse of duration τ, the corresponding collision time τ_c in the rest system of the atom (ion) decreases by a factor of γ; i.e., $\tau_c \sim \tau/\gamma$. Let us consider the possibility of observing in such experiments the inelastic processes accompanying the interaction between atoms and ultrashort electromagnetic pulses with values of relativistic factor $\gamma \sim 10^4$ attainable in modern heavy-particle accelerators [24] (these values correspond to the effective decrease in the pulse duration by four orders of magnitude). We will first obtain estimates for relativistic problems, in which the characteristic energy difference $\Delta E \sim mc^2$ (m is the electron mass and c is the velocity of light). The corresponding characteristic frequency is $\omega_a = mc^2/\hbar$; conse-

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quently, the characteristic times of a stationary target atom are

$$\tau_a = \frac{2\pi}{\omega_a} \approx 8.1 \times 10^{-21} \sim 10^{-20} \text{ s},$$

while the collision time in the rest system of the atom for femtosecond pulses of duration $\tau \sim 10^{-15}$ s attainable at present is $\tau_c \sim \tau/\gamma \sim 10^{-19}$ s. Thus, direct observation of the relativistic effects considered here requires that the pulse duration be reduced by an order of magnitude (i.e., to approximately 100 attos), which is in line with contemporary tendencies [1–3, 21].

In theoretical analysis of the effects accompanying the interaction of atoms with ultrashort electromagnetic pulses, a natural foundation for solving problems can be the sudden approximation, which is closely related [30] to the eikonal approximation and which was previously used only for solving nonrelativistic problems [31–35], in which the perturbation is not small enough for using perturbation theory, but the time of action of a perturbation is much shorter than the characteristic periods of time for an unperturbed system. This makes it possible to solve the problem without setting a limit on the perturbation intensity. The effects of interaction of atoms with ultrashort electromagnetic pulses can be attributed to such cases. Here, we apply the term ultrashort pulses to pulses with duration smaller than the characteristic times for the target atom, which can be in the ground state or in an excited state (including a highly excited Rydberg state) prior to the interaction. Such pulses may be of various origin [1–3, 36–39], but can also be the fields of heavy ions moving with a relativistic or ultrarelativistic velocity [22–26]. In the latter case, perturbation theory is inapplicable for the fields of ions with large charges [40] even for infinitely large energies of the ions. A nonrelativistic nonperturbative theory developed in [41] describes the electron transitions and radiation emitted by an atom during its interaction with a spatially inhomogeneous (over the target size) ultrashort electromagnetic pulse.

In this study, on the basis of the sudden approximation, we obtain a solution to the Dirac equation, which describes the behavior of a hydrogen-like atom during its interaction with a spatially inhomogeneous ultrashort electromagnetic pulse. The corresponding transition probabilities are expressed in terms of the known inelastic atomic form factors, which are widely used in the theory of relativistic collisions between charged particles and atoms. By way of example, we consider the inelastic processes accompanying the interaction between ultrashort pulses with hydrogen-like atoms; the probabilities of ionization and production of a bound–free electron–positron pair at a bare nucleus accompanied by the formation of a hydrogen-like atom in the final state and a positron in the continuum are calculated. The developed technique makes it possible to exactly take into account the spatial inhomogeneity (over the target size) of the ultrashort electromagnetic pulse as in the nonrelativistic theory [41]; however, in contrast to the nonrelativistic theory, this technique exactly takes into account the magnetic interaction also.

2. TRANSITION AMPLITUDE IN THE SUDDEN APPROXIMATION

In the terminology used in [31], the perturbation corresponding to the field of an ultrashort pulse is a shock of the scattering type. To illustrate the sudden approximation, it is obviously expedient to consider the formal solution of the Schrödinger equation (here and below, we use atomic units)

$$i\dot{\Psi} = (H_0 + U(t))\Psi, \tag{1}$$

where sudden perturbation $U(t)$ acts during a time much shorter than the characteristic time periods of an unperturbed system described by Hamiltonian H_0. In this case, in solving Eq. (1), we can disregard (during the time of action of perturbation $U(t)$) the evolution of the wavefunction under the action of intrinsic Hamiltonian H_0 and solve the equation

$$i\dot{\Psi} = U(t)\Psi.$$

It hence follows that

$$\Psi(t) = \exp\left\{-i\int_{t_0}^{t} U(t)dt\right\}\Psi(t_0). \tag{2}$$

Consequently, the amplitude of transition of a nonrelativistic atom from state $|i\rangle$ to state $|f\rangle$ as a result of sudden perturbation $U(t)$ has the form [31]

$$a_{if} = \langle f|\exp\left\{-i\int_{-\infty}^{+\infty} U(t)dt\right\}|i\rangle. \tag{3}$$

It can easily be seen that the same result can be obtained if we solve exactly Eq. (1) with a delta-shaped potential $\tilde{U}(t)$ connected with potential $U(t)$ via the relation

$$\tilde{U}(t) = U_0\delta(t), \quad U_0 = \int_{-\infty}^{+\infty} U(t)dt. \tag{4}$$

Precisely this circumstance will be used below for solving the Dirac equation in the sudden approximation.

The behavior of the electron in a hydrogen-like atom (with a nuclear charge Z_a on which no limitations are imposed except the applicability conditions [42] for the Dirac equation) in an external field

$$A^\mu = (\varphi, \mathbf{A})$$

will be described by the Dirac equation (the electron charge $e = -1$ at. unit)

$$i\dot{\Psi} = \left\{c\hat{\boldsymbol{\alpha}}\left(\hat{\mathbf{p}} + \frac{1}{c}\mathbf{A}\right) - \frac{Z_a}{r} - \varphi + \hat{\beta}c^2\right\}\Psi, \qquad (5)$$

where the terms

$$c\hat{\boldsymbol{\alpha}}\hat{\mathbf{p}} + \hat{\beta}c^2 - Z_a/r$$

are equal to the Hamiltonian H_0 of a single atom and the interaction of the atomic electron with the external field is described by the potential

$$U(t) = U(\mathbf{r}, t) = \hat{\boldsymbol{\alpha}}\mathbf{A} - \varphi,$$

where $\hat{\mathbf{p}}$ is the momentum operator, $\hat{\boldsymbol{\alpha}}$ and $\hat{\beta}$ are the Dirac matrices, and $\mathbf{r}$ are the coordinates of the atomic electron. We first choose the calibration of the electromagnetic wave potentials (vector potential $\mathbf{A}$ and scalar potential φ) so that the scalar potential is zero. We assume that the vector potential of the field of the wave is a function of coordinate $\mathbf{r}$ and time t,

$$\mathbf{A} = \mathbf{A}(\mathbf{r}, t) = \mathbf{A}(\eta),$$

where the phase of the wave is given by

$$\eta = \omega_0 t - \mathbf{k}_0 \cdot \mathbf{r};$$

here, wavevector $\mathbf{k}_0$ is such that

$$|\mathbf{k}_0| = \omega_0/c,$$

and ω_0 is the circular frequency. We carry out the gauge transformation [41]

$$\mathbf{A}' = \mathbf{A} + \nabla f, \quad \varphi' = \varphi - \frac{1}{c}\frac{\partial f}{\partial t},$$

where

$$f = -\mathbf{A} \cdot \mathbf{r}.$$

This gives

$$\mathbf{A}' = \mathbf{k}_0\left(\mathbf{r}\frac{d\mathbf{A}}{d\eta}\right), \quad \varphi' = -(\mathbf{E} \cdot \mathbf{r}),$$

where

$$\mathbf{E} = \mathbf{E}(\mathbf{r}, t) = -|\mathbf{k}_0|\frac{d\mathbf{A}}{d\eta}.$$

Consequently, in the new calibration, the vector and scalar potentials are connected through the relation

$$\mathbf{A}' = (\mathbf{k}_0/|\mathbf{k}_0|)\varphi'.$$

We assume that the z axis is directed along vector $\mathbf{k}_0$. In this case, the interaction of an atomic electron with the external field in Eq. (5) is given by

$$\begin{aligned} U(t) &= \hat{\boldsymbol{\alpha}}\mathbf{A}' - \varphi' \\ &= -\left(1 - \frac{\hat{\boldsymbol{\alpha}}\mathbf{k}_0}{|\mathbf{k}_0|}\right)\varphi' = -(1 - \hat{\alpha}_z)\varphi'. \end{aligned} \qquad (6)$$

We will operate with the new calibration and omit primes on the potentials. To solve the Dirac equation (5) in the sudden approximation, we write it in the form

$$i\dot{\Psi} = (H_0 + U(t))\Psi$$

and use the substitution (4) introduced at the beginning of this section. For this purpose, we introduce

$$\tilde{\varphi} = \varphi_0\delta(ct - z), \quad \varphi_0 = c\int_{-\infty}^{+\infty}\varphi dt, \qquad (7)$$

where $\varphi_0 = \varphi_0(\mathbf{r})$ (i.e., a function of only the coordinates $\mathbf{r}$ of the point of observation). Further, in accordance with formula (4), we replace $U(t)$ from formula (6) by

$$\tilde{U}(t) = -(1 - \hat{\alpha}_z)\tilde{\varphi}$$

or by

$$\tilde{U}(t) = U_0\delta(ct - z), \qquad (8)$$

where

$$U_0 = c\int_{-\infty}^{+\infty}U(t)dt = -(1 - \hat{\alpha}_z)\varphi_0. \qquad (9)$$

As a result, Eq. (5) assumes the form

$$i\dot{\Psi} = \left\{c\hat{\boldsymbol{\alpha}}\hat{\mathbf{p}} - \frac{Z_a}{r} + \hat{\beta}c^2 - (1 - \hat{\alpha}_z)\tilde{\varphi}\right\}\Psi. \qquad (10)$$

To obtain the exact solution to the Dirac equation with such a potential, we expand $\Psi = \Psi(\mathbf{r}, t)$ in the eigenfunctions $\phi_k(\mathbf{r})$ (with energies E_k) of the unperturbed atomic Hamiltonian

$$H_0 = c\hat{\boldsymbol{\alpha}}\hat{\mathbf{p}} + \hat{\beta}c^2 - Z_a/r.$$

This gives

$$\Psi(\mathbf{r}, t) = \sum_k a_k(t)\phi_k(\mathbf{r})\exp(-iE_k t).$$

Substituting this expansion into the left-hand side of the equation

$$i\dot{\Psi} = (H_0 + \tilde{U}(t))\Psi$$

and integrating, after premultiplying it by a state ϕ_f and taking into account the orthogonality of states ϕ_k, we obtain

$$\frac{da_f(t)}{dt} = -i\exp(iE_f t)\langle\phi_f|\tilde{U}(t)|\Psi(\mathbf{r}, t)\rangle. \qquad (11)$$

Let us suppose that the atom was in state ϕ_j before the collision; in this case, we have

$$\Psi(\mathbf{r}, t = -\infty) = \exp(-iE_j t)\phi_j(r), \qquad (12)$$

$$a_f(t = -\infty) = \delta_{fj}, \qquad (13)$$

where δ_{fj} is the Kronecker delta. Since

$$\tilde{U}(t) = U_0\delta(ct - z),$$

it is sufficient for solving Eq. (11) to know the values of $\Psi(\mathbf{r}, t)$ only for $ct = z$; these values can be determined from Eq. (10) as follows. We pass to the light-cone variables

$$\begin{aligned} z^- &= (ct - z), \\ z^+ &= (ct + z). \end{aligned} \qquad (14)$$

Retaining only the derivatives with respect to z^- in a small neighborhood of $z^- = 0$ and the singular potential $-(1 - \hat{\alpha}_z)\tilde{\varphi}$, we obtain the equation

$$ic(1 - \hat{\alpha}_z)\frac{\partial\Psi}{\partial z^-} = -(1 - \hat{\alpha}_z)\tilde{\varphi}\Psi. \qquad (15)$$

Since $\tilde{\varphi} = \varphi_0\delta(z^-)$, taking into account the relations

$$\begin{aligned} \frac{d}{dx}\theta(x) &= \delta(x), \\ \frac{d}{dx}\exp(\theta(x)) &= \delta(x)\exp(\theta(x)), \end{aligned} \qquad (16)$$

where

$$\theta(x) = \begin{cases} 0, & x < 0, \\ 1, & x > 0, \end{cases}$$

we obtain the solution to Eq. (15):

$$\begin{aligned} (1 - \hat{\alpha}_z)\Psi(z^- + 0) &= (1 - \hat{\alpha}_z) \\ &\times \exp\left[i\frac{\varphi_0}{c}\theta(z^-)\right]\Psi(z^- - 0). \end{aligned} \qquad (17)$$

Returning to time t and using conditions (12) and (13), we obtain a solution which is valid for $t < z/c$ and in the vicinity of $t = z/c$ (i.e., $t = z/c + \varepsilon$, $\varepsilon > 0$ and is small):

$$\begin{aligned} (1 - \hat{\alpha}_z)\Psi(\mathbf{r}, t) &= (1 - \hat{\alpha}_z) \\ \times \exp\left[i\theta(ct - z)\frac{\varphi_0}{c}\right] &\exp[-iE_j t]\phi_j(\mathbf{r}). \end{aligned} \qquad (18)$$

Substituting Eq. (18) into the right-hand side of Eq. (11) and integrating with respect to t taking into account initial conditions (13), we obtain

$$\begin{aligned} a_{fj} &= a_f(t = +\infty) \\ &= \delta_{fj} + i\int_{-\infty}^{+\infty} dt\exp(i(E_f - E_j)t) \\ &\times \langle\phi_f|\varphi_0\delta(z - ct)(1 - \hat{\alpha}_z)\exp\left[i\theta(ct - z)\frac{\varphi_0}{c}\right]|\phi_j\rangle. \end{aligned} \qquad (19)$$

Using relations (16), we obtain

$$\begin{aligned} a_{fj} &= \delta_{fj} + \langle\phi_f|(1 - \hat{\alpha}_z)\exp\left(i\frac{(E_f - E_j)z}{c}\right) \\ &\times \left[\exp\left(i\frac{\varphi_0}{c}\right) - 1\right]|\phi_j\rangle. \end{aligned} \qquad (20)$$

This is the required solution to the Dirac equation with potential

$$\tilde{U}(t) = U_0\delta(ct - z),$$

corresponding to the inclusion of potential (6) in the sudden approximation. The obtained expression can be written in a more convenient form by using the easily verifiable relation

$$\langle\phi_f|(1 - \hat{\alpha}_z)\exp\left(i\frac{(E_f - E_j)z}{c}\right)|\phi_j\rangle = \delta_{fj}. \qquad (21)$$

This gives

$$\begin{aligned} a_{fj} &= \langle\phi_f|(1 - \hat{\alpha}_z)\exp\left(i\frac{(E_f - E_j)z}{c}\right) \\ &\times \exp\left(i\frac{\varphi_0}{c}\right)|\phi_j\rangle. \end{aligned} \qquad (22)$$

To describe transitions in the case of the interaction of a complex multielectron atom with an ultrashort electromagnetic pulse, we proceed as follows. We assume that the states of atomic electrons are described as the products of one-electron wavefunctions and denote the energy of the electron with number a (where $a = 1, 2, \ldots, N$, N being the number of atomic electrons) by $E^{(a)}$ and the electron coordinates by $\mathbf{r}_a$. Then the natural generalization of amplitude (22) for a transition of the complex N-electron atom from the initial state $\phi_j = \phi_j(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ with energy

$$E_j = \sum_{a=1}^{N} E_j^{(a)}$$

to the final state $\phi_f = \phi_f(\mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_N)$ with energy

$$E_f = \sum_{a=1}^{N} E_f^{(a)}$$

has the form

$$a_{fj} = \langle\phi_f|\prod_{a=1}^{N}(1-\hat{\alpha}_z^{(a)})\exp\left[\frac{i}{c}\sum_{a=1}^{N}(E_f^{(a)}-E_j^{(a)})z_a\right] \times \exp\left(\frac{i}{c}\sum_{a=1}^{N}\varphi_0(\mathbf{r}_a)\right)|\phi_j\rangle, \qquad (23)$$

where matrix $\hat{\alpha}_z^{(a)}$ acts only on bispinor indices belonging to the atomic electron with number a.

3. TRANSITION PROBABILITIES

Let us consider the interaction of an atomic electron with a Gaussian electromagnetic pulse (with an effective duration on the order of λ^{-1}),

$$\mathbf{E}(\mathbf{r}, t) = \mathbf{E}_0\exp\left(-\lambda^2\left(t-\frac{\mathbf{k}_0\cdot\mathbf{r}}{\omega_0}\right)^2\right) \times \cos(\omega_0 t - \mathbf{k}_0\cdot\mathbf{r}), \qquad (24)$$

$$\varphi = -\mathbf{r}\cdot\mathbf{E}(\mathbf{r}, t), \quad \varphi_0 = c\mathbf{q}\cdot\mathbf{r},$$

where

$$\mathbf{q} = -\int_{-\infty}^{+\infty} dt\mathbf{E}(\mathbf{r}, t) = -\mathbf{E}_0\frac{\sqrt{\pi}}{\lambda}\exp\left(-\frac{\omega_0^2}{4\lambda^2}\right). \qquad (25)$$

Recollecting that

$$\hat{\alpha}_z = \hat{\boldsymbol{\alpha}}\mathbf{k}_0/k_0$$

and introducing the vector

$$\mathbf{Q} = (Q_x, Q_y, Q_z) = (q, 0, \Omega_{fj}/c),$$

where

$$\Omega_{fj} = E_f - E_j,$$

we write a_{fj} from relation (22) in the form

$$a_{fj} = \langle\phi_f|\left(1-\frac{\hat{\boldsymbol{\alpha}}\mathbf{k}_0}{k_0}\right)\exp(i\mathbf{Q}\cdot\mathbf{r})|\phi_j\rangle. \qquad (26)$$

Choosing now the z axis in the direction of vector $\mathbf{Q}$, we can write a_{fj} in the form

$$a_{fj} = \langle\phi_f|(1-\hat{\alpha}_z\cos\theta)\exp(iQz)|\phi_j\rangle - \langle\phi_f|\hat{\alpha}_x\sin\theta\exp(iQz)|\phi_j\rangle, \qquad (27)$$

where

$$Q = \sqrt{q^2+\Omega_{fj}^2/c^2}, \quad \cos\theta = \Omega_{fj}\Big/\sqrt{c^2q^2+\Omega_{fj}^2},$$

$$\sin\theta = q\Big/\sqrt{q^2+\Omega_{fj}^2/c^2}.$$

Thus, let us now suppose that the atom was in state ϕ_j with energy E_j prior to the interaction (i.e., at $t = -\infty$) with the field of an ultrashort pulse; then the probability of finding the atom in state ϕ_f with energy E_f after the interaction (i.e., for $t = +\infty$) is given by

$$|a_{fj}|^2 = \frac{q^2}{Q^2}\left(\frac{q^2}{Q^2}|F^{fj}|^2 + |G_x^{fj}|^2\right). \qquad (28)$$

Following [43–45], we have introduced the inelastic atomic form factors

$$F^{fj} = \langle\phi_f|\exp(iQz)|\phi_j\rangle = \frac{Qc}{\Omega_{fj}}\langle\phi_f|\hat{\alpha}_z\exp(iQz)|\phi_j\rangle,$$

$$G_x^{fj} = \langle\phi_f|\hat{\alpha}_x\exp(iQz)|\phi_j\rangle,$$

which are widely used in the theory of relativistic collisions of charged particles with atoms. Consequently, the above formulas make it possible [43–45] to determine the probabilities of excitation and ionization of a hydrogen-like atom interacting with an ultrashort electromagnetic pulse. These formulas can also be used for calculating the probability of production of an electron–positron pair during the interaction of a bare ion with an ultrashort electromagnetic pulse if we interpret this process as a transition of an electron from the states of the negative continuum (Dirac sea) to the states with a positive total energy of the hydrogen-like atom In all cases, we can use either the form factors (see, for example, [40, 45]) calculated using the so-called Coulomb–Dirac hydrogen-like wavefunctions for electrons and positrons, which leads to a complex numerical computation, or analytic expressions [40, 43–47] for the form factors determined with the Darwin quasi-relativistic wavefunctions and the Sommerfeld–Maue wavefunctions [40, 42, 44, 45, 48]. Strictly speaking, the quasi-relativistic functions are valid under the condition $Z_a \ll c$; if this inequality is violated, the results satisfactorily illustrate the behavior of the form factors qualitatively [40, 45].

The above formulas make it possible to calculate both the probabilities of inelastic processes having nonrelativistic analogs (excitation or ionization of an atom by an ultrashort electromagnetic pulse) and essentially relativistic effects (production of the electron–positron pairs). As in the nonrelativistic theory [41], the relativistic approach developed by us exactly takes into account the spatial inhomogeneity (over the target size) of the ultrashort electromagnetic pulse; however, unlike the nonrelativistic approach [41] (see also [5, 49]), it exactly takes into account the magnetic interaction also. Figures 1 and 2 show the ionization probabilities

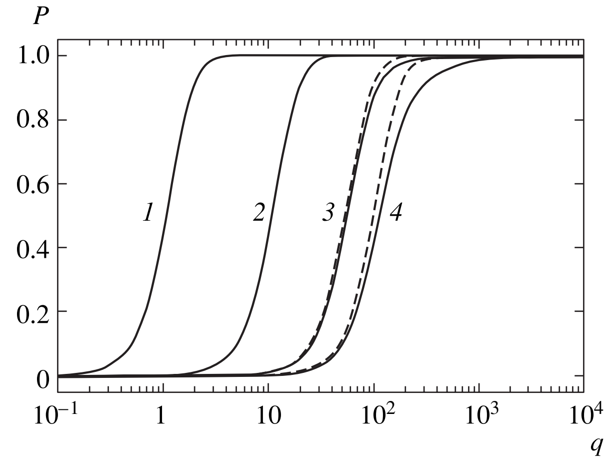

Fig. 1. Dependences of the ionization probability P (formation of a K vacancy) of hydrogen-like atoms for several values of nuclear charge Z_a on the transferred momentum (25) $q = |\mathbf{q}|$ (atomic units). The results of calculations for each value of Z_a are represented by two curves; the solid curve corresponds to relativistic calculations based on formula (28), while the dashed curve corresponds to the nonrelativistic calculation [41] (formula (28) for $c \longrightarrow \infty$) for $Z_a = 1$ (*1*), 10 (*2*), 50 (*3*), and 92 (*4*).

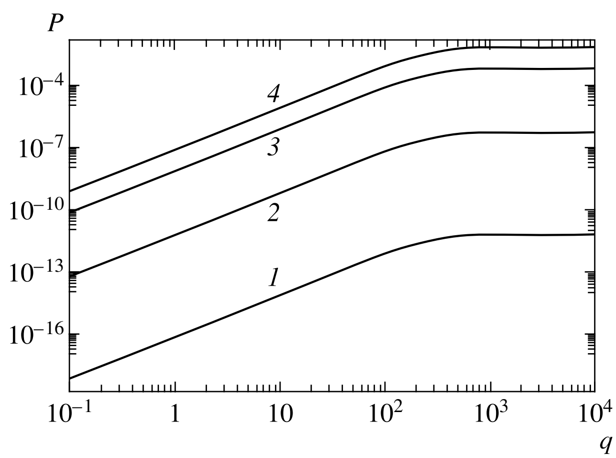

Fig. 2. Dependences of the probability P (28) of production of a free–bound electron–positron pair (the electron in the 1*s* state and the positron in the state of continuum of a hydrogen-like atom with effective atomic charge Z_a) on the transferred momentum (25) $q = |\mathbf{q}|$ (atomic units) for several values of $Z_a = 1$ (*1*), 10 (*2*), 50 (*3*), and 92 (*4*).

(the probabilities of formation of K vacancies) of hydrogen-like atoms and the probabilities of production of a free–bound electron–positron pair for several values of the nuclear charge Z_a. In our calculations, we used, following [40, 45], the quasi-relativistic Darwin wavefunctions for bound states and the Sommerfeld–Maue functions for the states of the continuum as the wavefunctions of the initial and final states; this makes it possible to calculate the form factors analytically.

To obtain the cross section of the transition of an atom from state ϕ_j with energy E_j to state ϕ_f with energy E_f, we must obviously multiply, according to [50], the probability $|a_{fj}|^2$ of the corresponding transition from formula (28) by the energy difference

$$\Omega_{fj} = E_f - E_j$$

and divide the result by the energy flux I equal to the integral of the absolute value of the Poynting vector

$$S(t) = c(4\pi)^{-1}\mathbf{E}^2$$

with respect to time, where $\mathbf{E}$ is expressed by formula (24). This gives

$$I = \int_{-\infty}^{+\infty} dt S(t) = \frac{c}{4\pi}\mathbf{E}_0^2 \frac{\sqrt{\pi}}{2\sqrt{2}\lambda} \times \left\{ \exp\left(-\frac{\omega_0^2}{2\lambda^2}\right) + 1 \right\}. \tag{29}$$

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